Material testing and hyperelastic material model curve fitting for Ogden, Polynomial and Yeoh models

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Outline

1. **Introduction**
   - Background information
   - Fitting hyperelastic material constants
   - Problem description and approach
2. **Materials and Models**
   - Material tests
   - Analytical equation for the Yeoh Model in tension/compression (example)
   - Optimisation problem
3. **Results and Discussion**
   - Ogden Model
   - Polynomial Model
   - Yeoh Model
4. **Conclusion**
   - General
   - Discrepancies with the Ogden Model implementation

Introduction
Background

- Static structural investigation on a silicone-suspended fracture plating system
- Modelling by means of finite element method (FEM)

$\Rightarrow$ How to model silicone in FEM?

Answer from literature: **hyperelastic material models**

Two types of hyperelastic material models[1]

Phenomenological models

material constants generated by curve fitting

Micro-mechanical models

material constants generated by specific material tests

The use of phenomenological model is suggested, as the physical significance of micro-mechanical material constants is often unclear[2].

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Fitting hyperelastic material constants

workflow and material model selection

1. **Material tests**: stress-stretch curves from e. g. uni- or bi-axial tensions tests, shear tests.
2. **Curve fitting**: adapting the material constants based on analytical models, to closely reproduce the measured curves.
3. **Validation** by modelling the material tests with FEM and comparison of the results.

Material models selected for this work:

- Ogden Model[3],
- Polynomial Model[4] (includes the Mooney-Rivlin Model[5;6]),
- Yeoh Model[7].

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Problem description and approach

FEM software ANSYS[a] offers higher-order hyperelastic material model input, but does not feature curve fitting for each of these models.

Approach:
Create a SCILAB[b] script and corresponding functions to conduct non-restrictive curve fitting.

[a] ANSYS Inc., Canonsburg, Pennsylvania, USA
[b] SCILAB ENTERPRISES, Versailles, France

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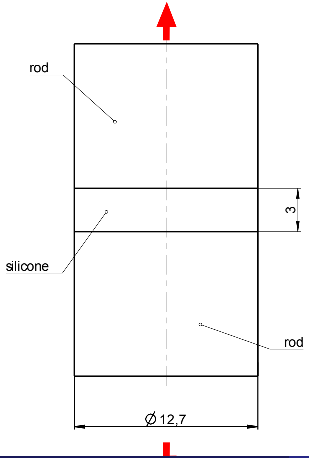
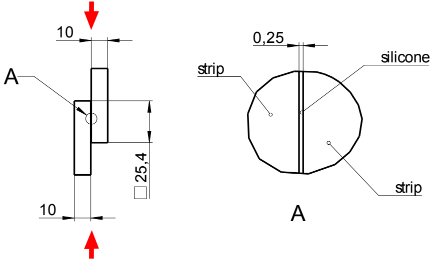
Materials and Methods

Material tests

Bonded compression/tension test (stress results to be corrected[8])



Simple shear test



all dimensions in Millimetres; "strip" refers to a steel strip

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Analytical equation of the Yeoh Model in tension/compression (example) I

1. Base equation of the Yeoh Model: $W = \sum_{i=1}^{n} C_i \cdot \left(I_1 - 3\right)^i$
   - W: strain energy,
   - C_i: material constant,
   - I_1: strain invariant,
   - n: model order.

2. strain invariants I_1 for compression/tension[9;10]:

$I_1 = \lambda^2 + 2\lambda^{-1}$,
$I_2 = \lambda^{-2} + 2\lambda$.

- λ: stretch due to tensile/compressive stress

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Analytical equation of the Yeoh Model in tension/compression (example) II

3. relation between engineering stress σ_e and stretch λ for an incompressible material under tension/compression[6;11]:
$$\sigma_e = 2 \cdot \left(\lambda - \lambda^{-2}\right) \cdot \left(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda} \cdot \frac{\partial W}{\partial I_2}\right).$$

4. Combining the aforementioned equations yields
$$\sigma_e(\lambda) = \sum_{i=1}^{n} 2 \cdot C_i \cdot i \cdot \left(\lambda - \lambda^{-2}\right) \cdot \left(\lambda^2 + 2\lambda^{-1} - 3\right)^{i-1}$$
for tension/compression of an n^{th} order Yeoh Model.

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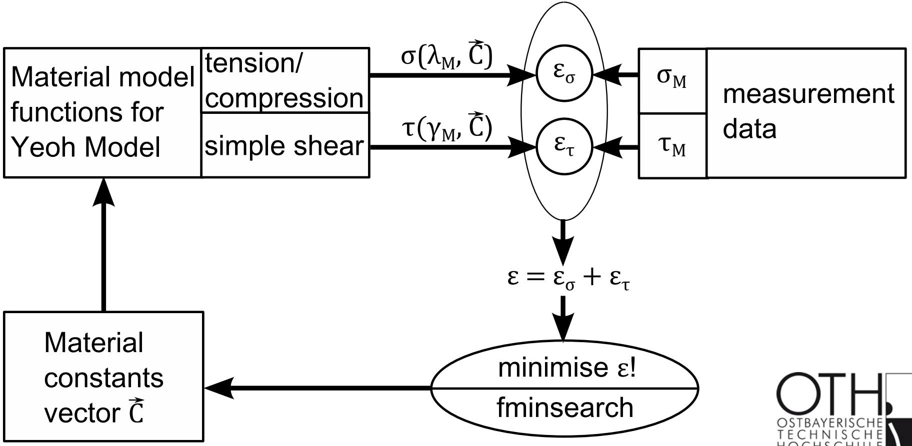
Optimisation problem
Scilab script overview

index “M” denotes data from measurement



Optimisation problem

notes

- Error criteria ϵ_σ and ϵ_τ may be normalised or unnormalised (absolute)

$$\epsilon_{norm} = \sum \left(1 - \frac{\sigma\left(\lambda, \vec{C}\right)}{\sigma_M\left(\lambda\right)} \right)^2$$

$$\epsilon_{abs} = \sum \left(\sigma_M\left(\lambda\right) - \sigma\left(\lambda, \vec{C}\right) \right)^2$$

- Adjusted coefficient of determination[12] was computed for goodness of fit evaluation
- Results for lower-order material models were compared to those from ANSYS

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Results and Discussion

Ogden Model; comparison with ANSYS

Ogden models used in curve fitting and results for the material parameters in ANSYS and using the SCILAB script ($[\mu_i]$ = MPa, $[\alpha_i] = 1$).

Order	error criterion	number of parameters	parameters from curve fitting routine: ANSYS	parameters from curve fitting routine: SCILAB script
2nd	norm.	4	$\mu_1 = -0.1244$ $\alpha_1 = 1.316$ $\mu_2 = 0.6225$ $\alpha_2 = 1.299$	$\mu_1 = 4.290$ $\alpha_1 = 0.1370$ $\mu_2 = 7.484 \times 10^{-3}$ $\alpha_2 = 4.346$
3rd	abs.	6	$\mu_1 = 0.1770$ $\alpha_1 = 1.055$ $\mu_2 = 0.2338$ $\alpha_2 = 1.041$ $\mu_3 = 0.3251$ $\alpha_3 = 0.9889$	$\mu_1 = 28.01 \times 10^{-3}$ $\alpha_1 = 3.525$ $\mu_2 = -4.385$ $\alpha_2 = 0.7777$ $\mu_3 = 7.270$ $\alpha_3 = 0.5449$
4th	abs.	8	not available	$\mu_1 = 58.90 \times 10^{-3}$ $\alpha_1 = 2.800$ $\mu_2 = -3.552$ $\alpha_2 = -0.3355$ $\mu_3 = 4.380$ $\alpha_3 = -79.13 \times 10^{-3}$ $\mu_4 = -0.4110$ $\alpha_4 = 0.9359$

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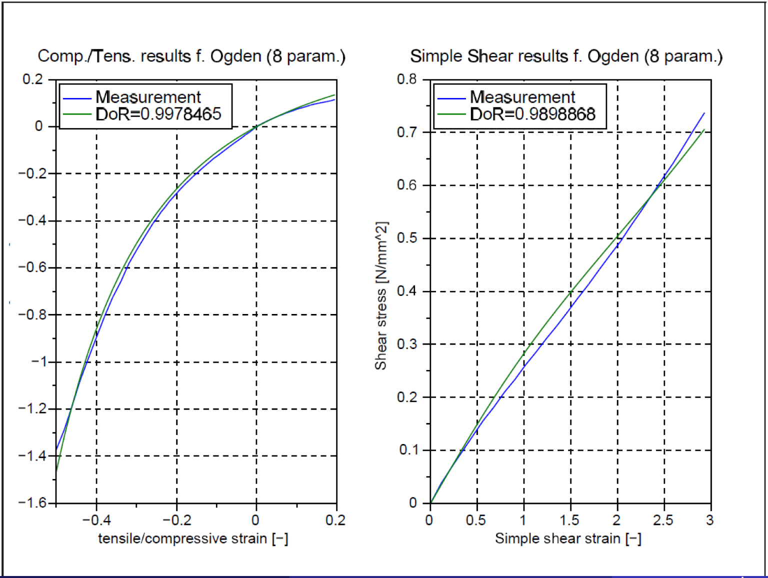
Results and Discussion

Ogden Model; curve fitting example



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Results and Discussion

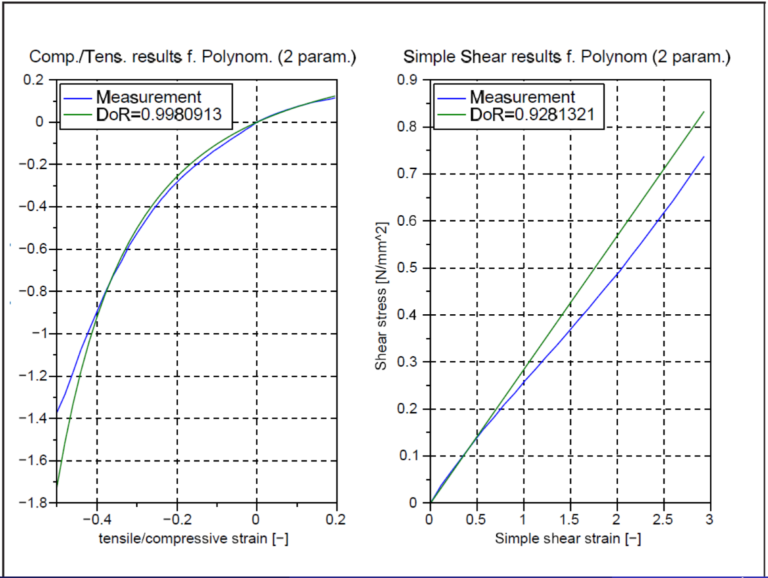
Polynomial Model; comparison with ANSYS

Polynomial models used in curve fitting and results for the material parameters in ANSYS and using the SCILAB script. The model with three parameters is equivalent to a three parameter Mooney-Rivlin model. ($[C_{ij}]$ = MPa).

Order	error criterion	number of parameters	parameters from curve fitting routine: ANSYS	parameters from curve fitting routine: SCILAB script
1st	norm.	2	$C_{10} = 37.98 \times 10^{-3}$ $C_{01} = 0.1043$	$C_{10} = 37.98 \times 10^{-3}$ $C_{01} = 0.1043$
-	abs.	3	$C_{10} = 74.92 \times 10^{-3}$ $C_{01} = 73.71 \times 10^{-3}$ $C_{11} = -1.972 \times 10^{-3}$	$C_{10} = 74.92 \times 10^{-3}$ $C_{01} = 73.71 \times 10^{-3}$ $C_{11} = -1.972 \times 10^{-3}$
2nd	abs.	5	$C_{10} = -88.41 \times 10^{-3}$ $C_{01} = 0.2197$ $C_{20} = 0.2122$ $C_{11} = -0.2909$ $C_{02} = 78.22 \times 10^{-3}$	$C_{10} = -88.41 \times 10^{-3}$ $C_{01} = 0.2197$ $C_{20} = 0.2122$ $C_{11} = -0.2909$ $C_{02} = 78.22 \times 10^{-3}$

Results and Discussion

Polynomial Model; curve fitting example



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Results and Discussion

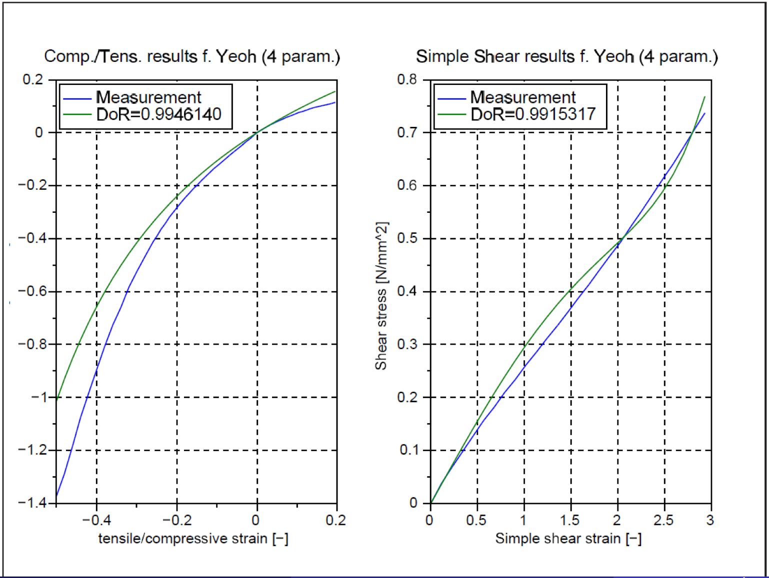
Yeoh Model; comparison with ANSYS

Yeoh models used in curve fitting and results for the material parameters in ANSYS and using the SCILAB script ($[C_{i0}]$ = MPa).

Order	error criterion	number of parameters	parameters from curve fitting routine: ANSYS	parameters from curve fitting routine: SCILAB script
3rd	norm.	3	$C_{10} = 0.1595$ $C_{20} = -6.930 \times 10^{-3}$ $C_{30} = 0.4030 \times 10^{-3}$	$C_{10} = 0.1595$ $C_{20} = -6.930 \times 10^{-3}$ $C_{30} = 0.4030 \times 10^{-3}$
4rd	norm.	4	not available	$C_{10} = 0.1593$ $C_{20} = -6.220 \times 10^{-3}$ $C_{30} = 0.2210 \times 10^{-3}$ $C_{40} = 11.90 \times 10^{-6}$
5rd	norm.	5	not available	$C_{10} = 0.1573$ $C_{20} = 2.391 \times 10^{-3}$ $C_{30} = -3.907 \times 10^{-3}$ $C_{40} = 0.6600 \times 10^{-3}$ $C_{50} = -32.60 \times 10^{-6}$

Results and Discussion

Yeoh Model; curve fitting example



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Conclusion

General

- Curve fitting script for fitting any order Ogden, any order Yeoh and second as well as first order Polynomial models, plus a three parameter Mooney-Rivlin Model.
- Yeoh and Polynomial Model curve fitting for lower-order models successfully validated against results from ANSYS
- SCILAB script allows for
  - curve fitting of higher order models than Ansys,
  - biasing of one of the material test results possible.
- higher order models tend to instabilities at small nominal-strains (ANSYS warning message)

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Conclusion

Discrepancies with the Ogden Model implementation I

- Ogden Model implementation $\Rightarrow$ difference in material constants between ANSYS and the SCILAB script
- Results for lower-order models from verification process:

Ogden models material parameters, fitted in ANSYS and using the SCILAB script ($[\mu_i] =$ MPa, $[\alpha_i] = 1$).

Order	error criterion	number of parameters	parameters from curve fitting routine: ANSYS	parameters from curve fitting routine: SCILAB script
1st	abs.	2	$\mu_1 = 0.7055$ $\alpha_1 = 1.0546$	$\mu_1 = 0.7208$ $\alpha_1 = 1.0345$
1st	norm.	2	$\mu_1 = 0.5571$ $\alpha_1 = 1.1609$	$\mu_1 = 0.6356$ $\alpha_1 = 1.023$

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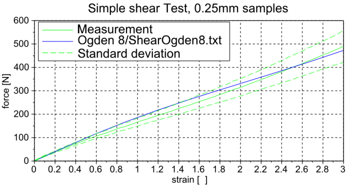
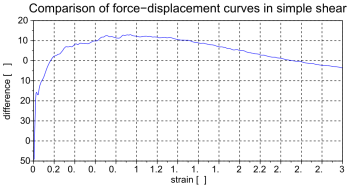
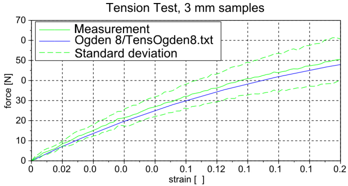
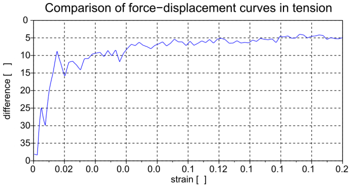
Conclusion

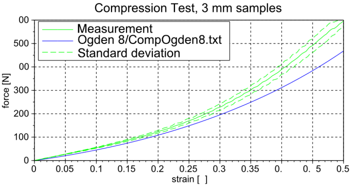
Ogden Model implementation II: FEA validation result

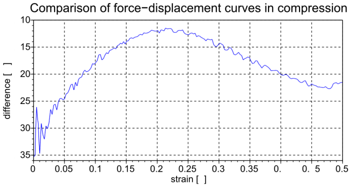












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Conclusion

Discrepancies with the Ogden Model implementation III

Ogden Model appears to be implemented correctly within the SCILAB script (cf. previous slide)

Differences could arise from

- the nonlinear nature of the problem,
- the fact that more than one minimum exists.

Besides: Even identical start values may lead to completely different material parameter sets[13].

Thank you!
Do you have questions or feedback?

References (two slides)

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